

Information thermodynamics for a multi-feedback process with time delay

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We investigate a measurement-feedback process of repeated operations with time delay. During a finite-time interval, measurement on the system is performed and the feedback protocol derived from the measurement outcome is applied with time delay. This protocol is maintained into the next interval until a new protocol from the next measurement is applied. Unlike a feedback process without delay, both memories associated with previous and present measurement outcomes are involved in the system dynamics, which naturally brings forth a joint system described by a system state and two memory states. The thermodynamic second law provides a lower bound for heat flow into a thermal reservoir by the (3-state) Shannon entropy change of the joint system. However, as the feedback protocol depends on memory states sequentially, we can deduce a tighter bound for heat flow by integrating out irrelevant memory states during dynamics. As a simple example, we consider the so-called cold damping feedback process where the velocity of a particle is measured and a dissipative feedback protocol is applied to decelerate the particle. We confirm that the heat flow is well above the tightest bound. We also examine the long-time limit of this feedback process, which turns out to exhibit an interesting instability transition as well as heating by controlling parameters such as measurement errors, time interval, protocol strength, and time delay length. We discuss the underlying mechanism for instability and heating, which might be unavoidable in reality.

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The recent information thermodynamics has been proven to resolve the paradox of Maxwell's demon [1] which was a long-lived problem in spite of enormous research works [1–5]. Replacing Maxwell's demon by a physical memory device, that was refined by Landauer [4], one is able to describe measurement inside a memory device and feedback after measurement acting on the system (engine) as thermodynamic processes. In the measurement process, information acquisition is realized as mutual information gain in the entropy of the joint system (system and memory device). In the subsequent feedback process, mutual information is expended through relaxation out of initial state producing work outside. The work production is balanced energetically by heat dissipation into the reservoir, which may be negative like in the Szilard engine [2], resulting in entropy loss in the reservoir. It was shown that such entropy loss in the reservoir, if any, be compensated sufficiently by the entropy gain of the joint system through mutual information decrease so as to satisfy the second law of thermodynamics. Hence the paradox of Maxwell's demon is resolved. It is the main feature of the information thermodynamics developed by Sagawa and Ueda [6–9]. The increase of the total entropy of the joint system and reservoir was proven with the aid of the fluctuation theorem (FT), which was discovered about two decades ago and has been regarded as a principle of nonequilibrium statistical mechanics [10–14]. The role of mutual information in feedback processes has also been confirmed in experiments [15, 16].

Memory is usually assumed to reach local equilibrium so fast that system state does not change during measurement. In a feedback process, system state changes

in time subject to a fixed protocol given from memory state picked out of its local equilibrium. In this sense, measurement and feedback can reasonably be regarded as processes with separated time periods [9, 18] and the fluctuation theorem for the total entropy production was shown to hold separately for the two bipartite periods [19].

In real situations, however, measurement process takes a finite time and the feedback protocol ought to be applied afterwards. This naturally generates time gap between the start of measurement and feedback. In the present work, we consider a realistic feedback process composed of multiple steps repeated in a finite-time interval, in each of which a feedback protocol is applied with time delay. As an example, we consider a simple cold-damping problem where the velocity of a particle is measured and a dissipative protocol is applied. In repeated feedback steps, the temperature of the system is expected to be cooled down below the reservoir temperature.

Consider that both system state $\mathbf{s}(t)$ and memory state $\mathbf{m}(t)$ in d dimensions coevolve in time t by their own dynamics. At measurement time $t = t_i$, memory starts to measure or copy $\mathbf{s}(t_i) = \mathbf{s}_i$ that acts as a protocol to drive memory into a copied state. One may think of the Langevin dynamics for such a process: $\dot{\mathbf{m}} = -\tau_m^{-1}(\mathbf{m} - \mathbf{s}_i) + \boldsymbol{\xi}(t)$ where $\langle \xi_a(t) \xi_b(t') \rangle = 2\tau_m^{-1}T_m \delta_{ab} \delta(t - t')$ with component indices $a, b = 1, \dots, d$ and temperature T_m of the reservoir surrounding memory. The Boltzmann constant is set to unity here and also in the following. Waiting for a long enough time δ compared to relaxation time τ_m , the memory reaches a local equilibrium with the conditional probability density function

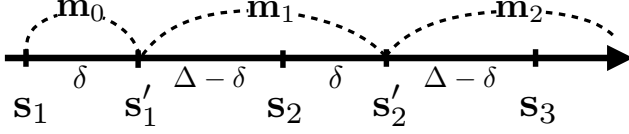


FIG. 1. (Color online) A schematic picture for repeated measurement-feedback processes. \mathbf{m}_i is a measurement outcome for an initial state \mathbf{s}_i of step i , which is applied as a protocol with time delay δ . This protocol is maintained into the next step until the next protocol is applied. $\mathbf{m}_0 = \mathbf{0}$ when there is no previous measurement.

(PDF) given as $p_{\mathbf{m}}(\mathbf{m}_i|\mathbf{s}_i) = (2\pi\sigma)^{-d/2}e^{-(\mathbf{m}_i-\mathbf{s}_i)^2/(2\sigma)}$ for $\sigma = \tau_{\mathbf{m}}T_{\mathbf{m}}$, which can be interpreted as measurement probability. For this period, the system undergoes a transition to state \mathbf{s}'_i at $t = t_i + \delta$ under a previous protocol \mathbf{m}_{i-1} . A new protocol \mathbf{m}_i chosen from the distribution $p_{\mathbf{m}}(\mathbf{m}_i|\mathbf{s}_i)$ is applied in turn to the dynamics of the system for $t_i + \delta < t < t_{i+1} = t_i + \Delta$. Since the measurement process at step i depends only on \mathbf{s}_i , as seen in the above Langevin equation, intermediate memory states between \mathbf{m}_{i-1} and \mathbf{m}_i can be averaged out for $t_i - (\Delta - \delta) \leq t < t_i + \delta$ without influencing the dynamics of $\mathbf{s}(t)$. In Fig. 1, the corresponding path of $\mathbf{s}(t)$ is shown with \mathbf{m}_{i-1} and \mathbf{m}_i coexisting in step i .

We introduce an adjoint dynamics with time-reverse protocols in which the probability of the system tracing the time-reverse path conjugate to a given (forward) path will be considered. The time-reversed path is defined as $\bar{\mathbf{s}}(t) = \varepsilon \mathbf{s}(t_N + t_1 - t)$ conjugate to a (forward) path $\mathbf{s}(t)$, where ε is the parity operator giving +1 (−1) if it is applied to a even (odd) parity state in time reversal such as position (momentum). The time-reverse protocols are defined as $\bar{\mathbf{m}}_i = \varepsilon \mathbf{m}_{N-i+1}$. For each of time-reverse protocols, not only the order in time is reversed, but also the parity is multiplied, copying a time-reverse state.

Let $\Pi_{\mathbf{s}_i, \mathbf{s}'_i}^{\mathbf{m}_{i-1}}[\mathbf{s}(t)]$ ($\Pi_{\mathbf{s}'_i, \mathbf{s}_{i+1}}^{\mathbf{m}_i}[\mathbf{s}(t)]$) be the conditional probability for a partial path from \mathbf{s}_i (\mathbf{s}'_i) to \mathbf{s}'_i (\mathbf{s}_{i+1}) under a protocol \mathbf{m}_{i-1} (\mathbf{m}_i) for $t_i \leq t < t_i + \delta$ ($t_i + \delta \leq t < t_{i+1}$) in step i . Similarly, we define the conditional path probabilities for time-reverse paths and protocols as $\Pi_{\varepsilon \mathbf{s}_{i+1}, \varepsilon \mathbf{s}'_i}^{\bar{\mathbf{m}}_i}[\bar{\mathbf{s}}(t)]$ and $\Pi_{\varepsilon \mathbf{s}'_i, \varepsilon \mathbf{s}_i}^{\bar{\mathbf{m}}_{i-1}}[\bar{\mathbf{s}}(t)]$. For usual thermodynamic process without feedback, the change in the total entropy of system and reservoir is known as the log-ratio of the path probabilities of the forward and time-reverse path. Extending to the joint system of system and memory, the corresponding *total entropy* change may be writ-

ten as

$$\begin{aligned} \sum_{i=1}^N \Delta S_{\text{tot},i} &= \prod_{i=1}^N \ln \left[\frac{\rho_i(\mathbf{s}_i) \rho_i(\mathbf{m}_{i-1}|\mathbf{s}_i) p_{\mathbf{m}}(\mathbf{m}_i|\mathbf{s}_i)}{\rho_{i+1}(\mathbf{s}_{i+1}) \bar{\rho}(\varepsilon \mathbf{m}_i, \varepsilon \mathbf{m}_{i-1}|\bar{\mathbf{s}}(t))} \right. \\ &\quad \times \left. \frac{\Pi_{\mathbf{s}_i, \mathbf{s}'_i}^{\mathbf{m}_{i-1}}[\mathbf{s}(t)] \Pi_{\mathbf{s}'_i, \mathbf{s}_{i+1}}^{\mathbf{m}_i}[\mathbf{s}(t)]}{\Pi_{\varepsilon \mathbf{s}_{i+1}, \varepsilon \mathbf{s}'_i}^{\bar{\mathbf{m}}_i}[\bar{\mathbf{s}}(t)] \Pi_{\varepsilon \mathbf{s}'_i, \varepsilon \mathbf{s}_i}^{\bar{\mathbf{m}}_{i-1}}[\bar{\mathbf{s}}(t)]} \right] \\ &= \sum_{i=1}^N [\Delta S_{\text{sm},i} + \Delta S_{\text{env},i}] \end{aligned} \quad (1)$$

where $\Delta S_{\text{tot},i}$ denotes the contribution from step i and ρ_i is the PDF at $t = t_i$. A conditional probability $\bar{\rho}$ for time-reverse protocols in the adjoint dynamics can be chosen in various ways, which will be discussed later.

The environmental entropy production for step i is defined as

$$\Delta S_{\text{env},i} = \ln \left[\frac{\Pi_{\mathbf{s}_i, \mathbf{s}'_i}^{\mathbf{m}_{i-1}}[\mathbf{s}(t)] \Pi_{\mathbf{s}'_i, \mathbf{s}_{i+1}}^{\mathbf{m}_i}[\mathbf{s}(t)]}{\Pi_{\varepsilon \mathbf{s}_{i+1}, \varepsilon \mathbf{s}'_i}^{\bar{\mathbf{m}}_i}[\bar{\mathbf{s}}(t)] \Pi_{\varepsilon \mathbf{s}'_i, \varepsilon \mathbf{s}_i}^{\bar{\mathbf{m}}_{i-1}}[\bar{\mathbf{s}}(t)]} \right]. \quad (2)$$

In the absence of odd-parity states, $\Delta S_{\text{env},i}$ is equal to Q_i/T for heat production Q_i into the reservoir at temperature T . However, it may contain an unconventional contribution due to an odd-parity force induced by an odd-parity protocol [20]. We will encounter this situation for a cold-damping problem where the velocity of a particle is measured.

$\Delta S_{\text{sm},i}$ is the entropy change of the joint system for step i , which reads $\Delta S_{\text{sys},i} - \Delta I_i$. Here ΔI_i is the mutual information change between system and memory. Note that the memory state does not change during each step. We find the first term to be the Shannon entropy change of the system, given as

$$\Delta S_{\text{sys},i} = -[\ln \rho_{i+1}(\mathbf{s}_{i+1}) - \ln \rho_i(\mathbf{s}_i)], \quad (3)$$

resulting from choosing the initial PDF of the time-reverse dynamics to be the final PDF $\rho_{i+1}(\mathbf{s}_{i+1})$ of the given dynamics. ΔI depends on how $\bar{\rho}(\varepsilon \mathbf{m}_i, \varepsilon \mathbf{m}_{i-1}|\bar{\mathbf{s}}(t))$ is chosen in the time-reverse dynamics.

We consider two choices in setting the distribution of protocols in the time-reverse dynamics, each of which yields mutual information as a part of $\Delta S_{\text{sm},i}$. The first one is given by

$$\bar{\rho}(\varepsilon \mathbf{m}_i, \varepsilon \mathbf{m}_{i-1}|\bar{\mathbf{s}}(t)) = \rho_{i+1}(\mathbf{m}_{i-1}, \mathbf{m}_i|\mathbf{s}_{i+1}), \quad (4)$$

which is the conditional PDF of the joint system at time t_{i+1} for the given dynamics found as $\rho_{i+1}(\mathbf{s}_{i+1}, \mathbf{m}_{i-1}, \mathbf{m}_i) \rho_{i+1}(\mathbf{s}_{i+1})^{-1}$. Then, we have

$$\Delta I_i^{(1)} = \ln \frac{\rho_{i+1}(\mathbf{m}_{i-1}, \mathbf{m}_i|\mathbf{s}_{i+1})}{\rho_i(\mathbf{m}_{i-1}|\mathbf{s}_i) p_{\mathbf{m}}(\mathbf{m}_i|\mathbf{s}_i)}, \quad (5)$$

which is the change in mutual information between system and two-state memory. The second choice is

$$\bar{\rho}(\varepsilon \mathbf{m}_i, \varepsilon \mathbf{m}_{i-1}|\bar{\mathbf{s}}(t)) = \rho_{i+1}(\mathbf{m}_i|\mathbf{s}_{i+1}) \rho_{i'}(\mathbf{m}_{i-1}|\mathbf{s}'_i, \mathbf{m}_i) \quad (6)$$

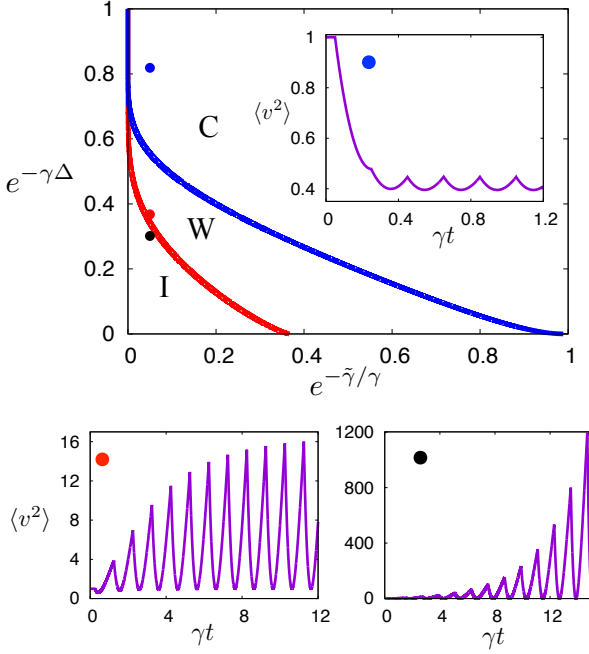


FIG. 3. (Color online) The diagram is drawn for $\delta/\Delta = 0.25$, $\sigma = 0.1$. C denotes the region for $T_\infty^{\text{av}} < T$, W for $T < T_\infty^{\text{av}} < \infty$, and I for $T_\infty^{\text{av}} = \infty$. The three points are picked from the three regions, for which $\langle v(t)^2 \rangle$ versus γt are shown.

where the superscript t denotes the transpose. Using the property of multi-variate Gaussian integral, the inversion of the matrix D_2 yields six moments such that

$$D_2^{-1} = \begin{pmatrix} \langle v_2^2 \rangle & \langle v_2 y_0 \rangle & \langle v_2 y_1 \rangle \\ \langle v_2 y_0 \rangle & \langle y_0^2 \rangle & \langle y_0 y_1 \rangle \\ \langle v_2 y_1 \rangle & \langle y_0 y_1 \rangle & \langle y_1^2 \rangle \end{pmatrix}, \quad (16)$$

which can be found in terms of T_1 , P_1 , and R_1 given in Eq. (12).

In particular, $T_2 = \langle v_2^2 \rangle$, $P_2 = \langle y_1^2 \rangle$, and $R_2 = \langle v_2 y_1 \rangle$ are found to satisfy the linear recursion relation:

$$\begin{aligned} T_2 &= w_\Delta + \sigma h^2 + K^2 T_1 + L^2 P_1 - 2KLR_1, \\ P_2 &= \sigma + T_1, \\ R_2 &= -\sigma h + KT_1 - LR_1, \end{aligned} \quad (17)$$

where $K = e^{-\gamma\Delta} - H$ with $H = (\tilde{\gamma}/\gamma)(1 - e^{-\gamma(\Delta-\delta)})$ and $L = (\tilde{\gamma}/\gamma)e^{-\gamma\Delta}(e^{\gamma\delta} - 1)$. The recursion relation can be rewritten as $\mathbf{Z}_2 = \mathbf{G} \cdot \mathbf{Z}_1 + \mathbf{A}$ for $\mathbf{Z}_i = (T_i, P_i, R_i)^t$ where the matrix \mathbf{G} and the vector \mathbf{A} are given from Eq. (17). $T_i = \langle v_i^2 \rangle$ is defined as the effective temperature at $t = t_i$ and is updated through feedback steps as $T_1 \rightarrow T_2 \rightarrow T_3 \rightarrow \dots$. The recursion relation will lead to a fixed value T_∞ only if $|\lambda_a| < 1$ for eigenvalues λ_a of \mathbf{G} for $a = 1, 2, 3$. The average effective temperature at step i can be found as $T_i^{\text{av}} = \Delta^{-1} \int_{t_i}^{t_{i+1}} dt \langle v(t)^2 \rangle$. Cold damping will be successful if $T_\infty^{\text{av}} < T$. In Fig. 3, C (cold) stands for the region for $T_\infty^{\text{av}} < T$, W (warm) for $T_\infty^{\text{av}} > T$, and I (instability) for the instability region with $|\lambda_a| \geq 1$.

We can compute the parts of the total entropy change. The Shannon entropy for $\rho(\mathbf{c}_2)$ in Eq. (15) can be written as $-\langle \ln \rho(\mathbf{c}_2) \rangle = (1/2)(-\ln \det D_2 + 3 \ln 2\pi + 3)$, and similarly for $\rho(\mathbf{c}_1)$. Then, we obtain $\langle \Delta S_{\text{sm}}^{(1)} \rangle = \langle \ln[\rho(\mathbf{c}_1)/\rho(\mathbf{c}_2)] \rangle$. By integrating $\rho(v(t), y_0, y_1)$ over y_0 or y_1 , one can find $\rho(v(t), y_i)$. Then, we find

$$\left\langle \ln \frac{\rho(v_1, y_0)}{\rho(v'_1, y_0)} \right\rangle = \frac{1}{2} \ln \left[e^{-2\gamma\delta} + \frac{w_\delta P_1}{T_1 P_1 - R_1^2} \right], \quad (18)$$

and

$$\begin{aligned} \left\langle \ln \frac{\rho(v'_1, y_1)}{\rho(v_2, y_1)} \right\rangle &= \frac{1}{2} \ln \left[e^{-2\gamma(\Delta-\delta)} \right. \\ &\quad \left. + \frac{w_{\Delta-\delta}(T_1 + \sigma)}{w_\delta T_1 + \sigma \langle v_1'^2 \rangle + H_\delta^2(T_1 P_1 - R_1^2)} \right], \end{aligned} \quad (19)$$

where $H_\delta = (\tilde{\gamma}/\gamma)(1 - e^{-\gamma\delta})$. Adding Eqs. (18) and (19) leads to $\langle \Delta S_{\text{sm}}^{(3)} \rangle$. $\langle \Delta S_{\text{sm}}^{(2)} \rangle$ in Eq. (7) can be found by adding the one in Eq. (19) and $\langle \ln \rho(\mathbf{c}_1) - \ln \rho(\mathbf{c}'_1) \rangle$. The three representations of the Shannon entropy change for the joint system are shown in Fig. 4.

The average environmental entropy production in Eq. (2) is given as

$$\begin{aligned} \langle \Delta S_{\text{env}} \rangle &= \left\langle \ln \left[\frac{\Pi_{v_1, v'_1}^{y_0}[v(t)] \Pi_{v'_1, v_2}^{y_1}[v(t)]}{\Pi_{-v_2, -v'_1}^{-y_1}[-v(t)] \Pi_{-v'_1, -v_1}^{-y_0}[-v(t)]} \right] \right\rangle \\ &= \frac{T_1 - T_2}{2T} - \frac{\tilde{\gamma}}{\gamma T} [\langle v_2 y_1 \rangle - \langle v'_1 y_1 \rangle] - \frac{\tilde{\gamma}}{\gamma T} [\langle v'_1 y_0 \rangle - \langle v_1 y_0 \rangle]. \end{aligned} \quad (20)$$

$\langle v'_1 y_1 \rangle$, and $\langle v'_1 y_0 \rangle$ can be obtained from $\langle v_2 y_1 \rangle$, and $\langle v_2 y_0 \rangle$ in Eq. (16) by putting $\Delta = \delta$.

The average heat production is found from $\int_{t_1}^{t_2} dt \langle [\gamma v(t) - \xi(t)] \circ v(t) \rangle$ with \circ denoting the Stratonovich calculus [25]. We find $\langle Q \rangle = \gamma \Delta (T^{\text{av}} - T)$. When the average effective temperature is lower than the reservoir temperature, meeting the need of cold damping, the average heat becomes negative, which is the situation in which the paradox of Maxwell's demon is raised.

$\langle \Delta S_{\text{uc}} \rangle = \langle \Delta S_{\text{env}} \rangle - \langle Q/T \rangle$ is an unconventional entropy production which is known to appear in the presence of an odd-parity force; $-\tilde{\gamma} y_i$ in our case [20]. Without feedback control, $\langle \Delta S_{\text{tot}} \rangle$ maintains positivity even for a negative $\langle Q \rangle/T$ thanks to $\langle \Delta S_{\text{uc}} \rangle$. For feedback process, $-\langle \Delta I \rangle$ plays an additive role in compensating entropy loss in reservoir together with $\langle \Delta S_{\text{uc}} \rangle$.

In Fig. 4, we display the components comprising the total entropy change at the fixed point of the recursive feedback process for $T_1 = T_2$ in the above equations. In the figure, $\langle \Delta S_{\text{sm}}^{(\alpha)} + \Delta S_{\text{uc}} \rangle$ is shown to be greater than $-\langle Q/T \rangle$ for all α , which confirms the generalized second law of thermodynamics. As expected from Fig. 2, $\langle \Delta S_{\text{tot}}^{(3)} \rangle$ is shown to yield the tightest bound.

We examine the generalized thermodynamic second law in the presence of coexisting past and present memories. We show the total entropy change to have the tightest bound as only mutual informations influencing

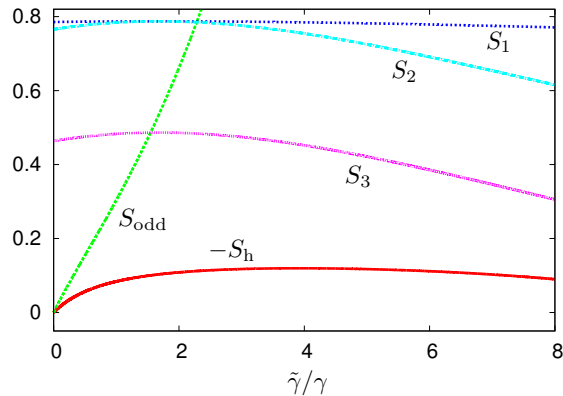


FIG. 4. (Color online) The components of ΔS_{tot} as functions of $\tilde{\gamma}/\gamma$ at the fixed point in the recursion procedure where $T_i = T_{i+1}$. Here, the plot is drawn for $\sigma = 0.1$, $\gamma\Delta = 0.2$, $\gamma\delta = 0.05$, and $T = 1$. For simplicity, we use $S_i = \langle \Delta S_{\text{sm}}^{(\alpha)} \rangle$ for $\alpha = 1, 2, 3$, $S_{\text{odd}} = \langle \Delta S_{\text{uc}} \rangle$, and $S_h = \langle Q/T \rangle$.

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